The Modern Subject and the Problem of Mathematics

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Abstract

This research forms part of a larger book project examining how the meanings and values ascribed to mathematics—as a universal, neutral discourse and as an idiom of reason and truth—came into being and about its cultural circulation between the 1920s and 1960s in American colleges and universities. Drawing on publications and sources from institutional archives such as the Rockefeller Archive Center, this project explores the relations and exchanges between mathematicians and scholars across the arts and humanities over what knowing mathematics entailed and what it meant to be modern.

Making Mathematics Modern

It might be tempting to think of mathematics in ahistorical and romanticized terms, of a discipline whose subject matter can be conceived of as beautiful, eternal, and defined by nature. But in September 1964, *Scientific American* published a special issue pointing to a more situated and contextualized understanding, one that examined "[m]athematics in the Modern World." With contributions from several leading mathematicians in the United States—including Richard Courant, Mark Kac, and Philip Davis—the issue sought to convey the depth and breadth of mathematical research. Elaborating on the concepts of "number," "geometry," algebra," and "probability" as fields of abstract inquiry within mathematics, the authors also lauded the introduction of mathematical theories and techniques to other domains of knowledge since World War II, including the humanities and the human sciences.[1] But the widening scope and nature of mathematical knowledge also raised concerns: If mathematics comprised diverse fields of study and increasing areas of application, what, if anything at all, held mathematics together? How should mathematics be characterized as a field or discipline?

The problem of defining and describing mathematics, argued the introductory article of the special issue, derived from institutional and intellectual transformations in the subject that had unfolded around the turn of the twentieth century. Professional mathematical organizations in the US, for instance, had witnessed unprecedented growth since the 1900s, with membership numbers increasing thirty-fold.[2] Likewise, American colleges and universities over the same period had experienced a similar increase in the number of undergraduates majoring in mathematics and graduate students pursuing doctoral studies.[3] For mathematicians like Richard Courant, such growth came on the heels of renewed interest in the epistemological foundations of mathematics and the emergence of new fields of research such as topology and group theory. These new fields, Courant observed, reflected a tendency towards formalism, abstraction, and general theory. They also reflected preferences for the treatment of mathematical theories as logically consistent systems and worlds of their own. Despite the inclination toward "progressive abstraction, logically rigorous axiomatic

deduction and ever wider generalization," Courant maintained that the "essence" of mathematics lay in "[t]he interplay between generality and individuality, deduction and construction, logic and imagination."[4]

The editors of the special issue selected a curious image to represent and reinforce Courant's vision of mathematics as a modern subject: René Magritte's *La lunette d'approche*. Originally painted in 1963, the surrealist painting depicts a vista that both is and is not discernable through a window. While the image makes no overt reference to either mathematics or mathematicians, the editors of *Scientific American* reasoned that the painting's elusive subject matter embodied the abstract nature of mathematics characterized in the special issue. "The painting," they elaborated, "symbolizes those aspects of mathematics which makes outrageous new assumptions to erect new systems."[5] By placing Magritte's work on the cover, the editors intimated a "modern" affinity between mathematics and the arts that saw both as endeavors grounded in human creativity rather than nature.

Historians and mathematicians alike have understood the mid-century relations drawn between mathematics and the arts to reflect the dual nature of the subject as a humanistic and scientific discipline. Between World War II and the 1970s, new fields of mathematical inquiry proliferated in the human sciences through new networks of government and philanthropic funding, seminars, institutes, and conferences that complemented existing university departments, academic journals, and professional societies. Such developments were also interpreted to be a direct consequence of "mathematical modernism," in which, between 1890 and 1930, mathematics was reconceived around notions of abstraction, general theory, and formalism, and was understood to deal with "made," disembodied concepts.[6] The modernist transformation of mathematics was successful, argues Jeremy Gray, "because it connected fruitfully with what mathematicians were doing and with the image they were creating for themselves as an autonomous body of professionals within, or alongside, the disciplines of philosophy and science."[7] Here, mathematical modernism emerged as a separate but parallel development to concurrent transformations in the arts and humanities, and was assumed to develop according to an internalist, disciplinary logic. By the end of

the Cold War in the US, the dual nature of mathematics seemed to have crystallized. Institutions for applied mathematics emerged by associating the rhetoric of autonomy and the comparison with the creative arts with its "modernist transformation," and by recognizing and advocating the extension of mathematics to broader domains in the technical, applied, and human sciences.[8]

Challenging the analytics that structure readings of modern mathematics, my book project describes how the conceptual categories of "modern," "creative," and "autonomy" developed as markers of social belonging that proliferated and culturally circulated in twentieth-century US intellectual life. This project emphasizes a critical perspective upon how migration, institutional practices, and exchanges produced them as categories of social existence and demarcation. At the turn of the twentieth century, artists and designers encountered new mathematical ideas and techniques as resources for describing, analyzing, and creating new forms of art. Likewise, mathematicians joined critics' and art historians' discussions over the meanings of art, craft, and aesthetics. Such exchanges occurred within interstitial spaces that reflected the shifting institutional organization of American higher education. Providing an account of contemporaneous efforts in mathematics to engage the arts and humanities, I not only show how such exchanges equally contributed to an understanding of mathematics as both scientific and humanistic, but also illuminate the concrete connections among the fields in twentieth-century US intellectual life.

By recovering this alternative history, I point out that when historians interpret modern mathematics as an intellectual enterprise that is simply separate and parallel to the arts, we miss a far more complex historical drama. By remaining within the epistemological frame of disciplinary autonomy, scholars inadvertently take for granted how mathematicians have rethought the characterization and meanings of their subject in relation to other disciplines. In other words, scholars take as evident the subject matter of each intellectual field as already being known, thereby naturalizing claims about the differences and similarities in their intellectual dynamics. Consequently, they foreclose the possibility of examining those assumptions, practices, and institutional infrastructures that enabled such understandings in the first place.[9] Recovering this history is crucial, particularly

at a moment when the value of the arts, humanities, sciences, and higher education more generally, are being reevaluated and scrutinized anew.

Mathematics in US Intellectual Life

Sources drawn from the Rockefeller Archive Center facilitated the analysis of several case studies alongside international policies and demographic shifts in the mathematical profession. In nineteenth-century institutions of higher learning in the United States, mathematics was valued primarily for its role in education and for its presumed exemplification of the highest form of reason. Mathematicians' efforts centered on supplementing the liberal arts curriculum with courses in calculus, geometry, and algebra.[10] By the first few decades of the twentieth century, mathematicians and mathematical work occupied a different role. No longer valued just for their ideas and ideals about mathematics in education, mathematicians were to be recognized as members of a research community whose recognition derived from their contributions to the methods and subject matter of mathematics itself.

On one level, such shifts were embedded within the institutional remaking of American higher education and the emergence of research universities that no longer revolved around a core set of collegiate values. Between 1890 and 1920, institutions such as Harvard, Columbia, Johns Hopkins, and the University of Chicago increasingly constituted a diverse array of colleges, professional schools, laboratories, liberal arts departments, museums, and observatories.[11] Academic researchers relied on private industry and philanthropic foundations like the Rockefeller Foundation and Carnegie Corporation for monetary support.[12] American universities also encompassed the development of what Joel Isaac has called the "interstitial academy," an assortment of university seminars, reading groups, and other enclaves through which to discuss or exchange ideas outside of established departments.[13]

On another level, transformations in the field and professionalization of mathematics were also rooted in international developments and several

initiatives in international education. In 1919, a nongovernmental international education organization was founded that would attain the broadest reach through its close ties to wealthy corporate philanthropists and State Department insiders: the Institute of International Education (IIE). Supported by the Rockefeller Foundation and the Carnegie Corporation, the IIE offered professional services for international educational programs, such as the administration of fellowships for foundations and governments, the coordination of international education and training programs, and the generation of educational information for foreigners and Americans. The IIE directed its efforts at educating foreign elites as a tool of US foreign relations outreach. Against this backdrop, mathematicians trained in the United States saw an increase of students not just from Europe, but also from China. Because they were experiencing "cultural renaissance" during the 1920s and 1930s, East Asia and the Middle East were of particular interest. Despite immigration quotas and isolationist policies, China's revolutionary upheavals drew the attention of American educators as an opportunity to enact greater transformations, including Harvard mathematician George D. Birkhoff.[14] For mathematicians such as Birkhoff, the proliferation of modern mathematics and its internationalization bolstered the value of the subject as an autonomous, creative subject.

The International Education Board's annual reports offer insight into the dynamics of migration and mathematical exchange. Established in January 1923, this Rockefeller philanthropy was founded "for the purpose of cooperating with foreign institutions and agencies engaged in the conduct and promotion of education." It also established science and mathematics as a focus:

> Scientific progress is a world-wide movement. A step forward is made here, another there. In incalculable ways, suggestions and discoveries originating at different points are brought together to achieve results which no one could have predicted or imagined. It is therefore extremely important that advanced workers have knowledge of one another's problems, methods, and results; that young men now in training who give promise of substantial development should, in their formative period, enjoy the stimulus to be derived from contact with productive scientists in other countries.

Emphasizing this philosophy, the IEB offered traveling fellowships not just for physics, chemistry, and biology, but also for mathematics.[15]

My research at the RAC included reviewing records and grant applications in which mathematics was singled out for application in the humanities and social sciences.[16] One of the case studies my book project examines is the production of mathematician George D. Birkhoff's theory of aesthetic measure at Harvard between the 1920s and 1930s. As a formal and quantitative technique for assessing artistic forms, Birkhoff's theory of aesthetic measure drew from decorative designs that have been considered "antimodern," developed out of the mathematics classroom, and found new life in design curricula. His work stemmed out of a project intended to examine the "Internationalization of the Mathematical Bases of the Art," and in turn to explore whether "aesthetic judgments about art could be made through objective and mathematical means."[17] Funding he received from the project enabled him to travel across various parts of Europe and Asia—including India, Thailand, China, Egypt, and Hungary-- to collect, catalogue, and analyze the whole range of artistic forms that were specific to a particular culture. From one country to the next, Birkhoff planned to dissect the literatures that described how "past and present practices and theories in art" had helped form new kinds of artistic creations, whether paintings, sculptures, musical compositions, or literary texts. Knowing that background was important, but only up to a certain point. A more crucial and essential task was to derive the particular order, arrangement, and set of elements undergirding a specific piece of art in the first place. Birkhoff set out to mathematically define and "give a more mature formulation" of art forms in systematic terms.[18]

His larger ambition, as he later explained in retrospect, was to demonstrate and, if possible, to prove, the necessity of "purely mathematical thought forms" for grasping those qualities of everyday life that remained elusive and therefore indescribable: "the subjective."[19] Extracting the inner thoughts and uncovering the mechanisms behind feelings of "aesthetic pleasure" within individuals posed a problem that Birkhoff wanted to resolve. Approaching this task, however, required answering several questions first: how could one draw on mathematicians' "skillful work" on the "modification of abstractions" to elucidate

"aesthetic pleasure"? [20] If so, in what ways could the "formal principles of mathematics be used to analyze artistic elements and determine their subjective, aesthetic values"? [21] His mathematical work broadly questioned the place of mathematics within and across all domains of inquiry. Following a series of activities, including giving regular presentations at mathematics conferences, and offering courses to explore the "mathematical elements of the arts," Birkhoff consolidated his thoughts and findings into a book published by Harvard University Press in 1934. Titled, *Aesthetic Measure*, he introduced a theoretical and quantitative approach to turn the private, individuated, and inexpressible act of judging art into a consistent, reliable, and objective assessment of what a viewer's response to art was.[22]

Focusing on how Birkhoff's development of aesthetic measure as both a theory and quantitative technique to evaluate aesthetic judgments emerged and related to concurrent developments in physiological aesthetics and design theory, I examined his correspondence with physician and administrator Simon Flexner, first director of the Rockefeller Institute for Medical Research and a Rockefeller Foundation trustee. By aesthetic measure, Birkhoff's interest lay in the problem of reliably measuring a viewer's emotional response to an art form. Aesthetic judgments of this kind had previously seemed impossible to collect, either because viewers struggled to verbally express their inner responses or because viewers' descriptions of their responses varied to a degree that they could not be reasonably compared. In Birkhoff's view, a reliable measure of aesthetic value (M) could be made by joining knowledge about human perception drawn from physiological aesthetics to a logical and formal mode of reasoning drawn from mathematics. In his words:

> The typical aesthetic experience may be regarded as compounded of three successive phases: (I) a preliminary effort of attention, which is necessary for the act of perception, and which increases in proportion to what we shall complexity (C) of the object; (2) the feeling of value or *aesthetic measure* (M) which rewards this effort; and finally (3) a realization that the object is characterized by a certain harmony, symmetry, or *order* (O), more or less concealed, which seems necessary to the aesthetic effect.[23]

Stated alternatively, Birkhoff contended that in the resulting equation M = O/C, the measure of aesthetic quality—or a viewer's emotional response—depended upon the density of order relations in an aesthetic object. Aesthetic quality was understood to be inversely proportional to the amount of attention required to wholly perceive an object. Simon Flexner's correspondence with Birkhoff reflected enthusiasm for an affinity between physiology and aesthetics bridged by mathematics.

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[1] The special issue's table of contents reflects this dual focus and organization. In *Scientific American* 211 (1964): 3.

[2] The New York Mathematical Society was founded in 1888 and had sixteen members at its inception. Within two years, membership ballooned to 210. In Karen Parshall and David Rowe, *The Emergence of the American Mathematical Research Community, 1876-1900: J. J. Sylvester, Felix Klein, and E. H. Moore* (London: London Mathematical Society, 1994), 336.

[3] Before 1875, American universities had conferred a total of only six degrees in the field. During the next fifteen years, thirty-nine Americans took doctorates in the US, and another fifteen earned their degrees abroad. These figures were dwarfed again by those of the final decade of the century, which witnessed a total of 107 new Ph.D.s in mathematics, 85 of which were earned in the US. R.G.D. Richardson, "The Ph.D. Degree in Mathematical Research," *American Mathematical Monthly* 43 (1936): 199-215.

[4] Richard Courant, "Mathematics in the Modern World," *Scientific American* 211 (1964): 42-43.

[5] Ibid., 4.

[6] Morris Kline, *Mathematical Thought from Ancient to Modern Times*, Volume 3 (New York: Oxford University Press, 1999), 1032.

[7] Jeremy Gray, *Plato's Ghost: The Modernist Transformation of Mathematics*, (Princeton, NJ: Princeton University Press, 2008), 3.

[8] Ibid., 1. Mathematicians who corroborate this image of mathematics included "Paul Halmos, "Mathematics as a Creative Art," *American Scientist* 56 (1968): 375-389; Marshall Stone, "The Revolution in Mathematics," *American Mathematical Monthly* 68 (1961): 715-734. For a historical account of the 1960s emergence of "the mathematical sciences" as a new term of art to reflect this dual nature, see Alma Steingart, "Conditional Inequalities: American Pure and Applied Mathematics, 1940-1975" (Ph.D. diss, MIT, 2011), especially 201-262.

[9] In making this argument, I draw from the arguments in Joan Scott, "The Evidence of Experience," *Critical Inquiry* 17, no. 4 (Summer, 1991).

[10] Parshall and Rowe, *The Emergence*; Karen Parshall, "Defining a Mathematical Research School: The Case of Algebra at the University of Chicago, 1892–1945," *Historia Mathematica* 31 (2004): 263-278.

[11] See Laurence R. Veysey, *The Emergence of the American* University (Chicago: University of Chicago Press, 1965); Jonathan R. Cole, *The Great American University: Its*

Rise to Preeminence, Its Indispensable National Role, and Why it Must be Protected (New York: Public Affairs, 2009), Julie Reuben, *Making of the Modern University: Intellectual Transformation and the Marginalization of Morality* (Chicago: University of Chicago Press, 1996). Frederick Rudolph, *The American College and University: A History* (Athens, GA: University of Georgia Press, 1990), esp. pp. 462-464; Roger L. Geiger, *To Advance Knowledge: The Growth of American Research Universities, 1900-1940* (New York: Oxford University Press, 1986), especially pages 1-57.

[12] Robert E. Kohler, *Partners in Science: Foundations and Natural Scientists, 1900-1945* (Chicago: University of Chicago Press, 1991). On the history of relations between mathematics and philanthropic institutions, see Reinhard Siegmund-Schultze, *Rockefeller and the Internationalization of Mathematics Between the Two World Wars* (Basel: Birkhäuser Verlag, 2001).

[13] Joel Isaac, *Working Knowledge: Working Knowledge: Making the Human Sciences from Parsons to Kuhn* (Cambridge, MA: Harvard University Press, 2012), 32-36.

[14] See Stephen Duggan, "The Foreign Student and the Immigration Law," *Fourth Annual Report of the Director* (New York: Institute of International Education, 1923), 1-3. The Rockefeller Foundation would fund many projects benefiting the Westernization of Chinese educational infrastructure, including aid to Chinese students in the United States.

[15] IEB Annual Report, February 3, 1923—June 30, 1924, International Education Board Records, RAC, 6-8.

[16] In particular, the "Application of Mathematics to Social Sciences" at Dartmouth College in FA733, and the significance of mathematical theories to the Committee on Social Thought in FA732H, Ford Foundation Records, RAC.

[17] George D. Birkhoff, "Proposed Research: The Internationalization of the Mathematical Bases of Art as Shown in Form, Color and Sound," December 1926, Reel 109, Series 1, FA746, Simon Flexner – APS Microfilm Collection, RAC.

[18] Ibid.

[19] George D. Birkhoff, "Mathematics: Quantity and Order" *Science Today* (1934): 293-317.

[20] Ibid.

[21] Ibid.

[22] George D. Birkhoff, *Aesthetic Measure* (Cambridge, MA: Harvard University Press, 1934).

[23] Birkhoff, *Aesthetic Measure*, 3-4.